Inequality with logarithms and exponents.

https://www.linkedin.com/feed/update/urn:li:activity:6726861219304673281

Let a and b be two real numbers; both greater than 1. Prove that

$$(a^x b^y)^{1/\ln(ab)} \le \frac{e^x \ln a + e^y \ln b}{\ln(ab)}$$

Solution by Arkady Alt, **San Jose** ,**California**, **USA.**

By replacing (p,q) in inequality* $e^{px+qy} \le pe^x + qe^y, p,q \ge 0, p+q = 1$ with $\left(\frac{\ln a}{\ln(ab)}, \frac{\ln b}{\ln(ab)}\right)$ we obtain $\frac{e^x \ln a + e^y \ln b}{\ln(ab)} \ge e^{\frac{x\ln a + y \ln b}{\ln(ab)}} = (a^x b^y)^{1/\ln(ab)}$.

* Since e^x is concave up function then $e^{px+qy} \le pe^x + qe^y$ for any $p,q \ge 0, p+q = 1$.
or, applying Weighted AM-GM Inequality $u^p v^q \le pu + qv$ to $(u,v) = (e^{q(x-y)}, e^{p(y-x)})$ we obtain $(e^{q(x-y)})^p \cdot (e^{p(y-x)})^q \le pe^{q(x-y)} + qe^{p(y-x)} \iff 1 \le pe^{q(x-y)} + qe^{p(y-x)} \iff$
$e^{px+qy} \le (pe^{q(x-y)} + qe^{p(y-x)})e^{px+qy} = pe^x + qe^y$.